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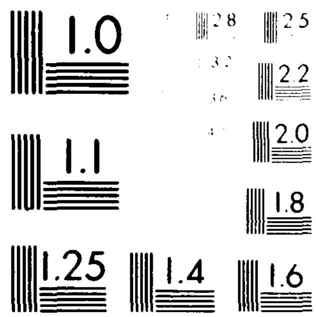

MICROCOPY RESOLUTION TEST CHART

IMPROVED METHODS FOR COMPUTING DRAG CORRECTED MISSILE IMPACT PREDICTIONS IN REAL TIME

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1. Introduction. During missile flight tests at White Sands Missile Range (WSMR), position data on the current status of the missile is transmitted 20 times per second from radar sites to a Univac 1108 computer. Consecutive pairs of such data are averaged 10 times per second and computations for plotting displays such as current position, range verses altitude, or impact prediction are based upon this averaged data.

In the event that a missile veers from its planned trajectory, it will be necessary to terminate thrust to prevent the missile from impacting in a populated area. For this reason, the Range Safety Officer (RSO) requires that for each computational cycle (10 per second) an instantaneous impact prediction (IIP) of the missile be computed. This point is the intersection of the missile trajectory, should thrust be terminated, with the Clarke Spheroid (of 1866) model of the Earth at an altitude of 4000 feet.

If the effects of atmospheric drag are neglected then a vacuum IIP can be rapidly computed using Kepler's central force equations, since the missile trajectory is an ellipse. This method is applicable for a variety of low drag vehicles whose exit from and reentry to the Earth's atmosphere occur at high angles. Since approximately one millisecond is required to compute a vacuum IIP, it is always computed at a 10 per second rate. Details of this computation are given in [1].

For low altitude missiles, whose trajectory is significantly affected by atmospheric drag, a drag-corrected IIP calculation

is also required by the RSO. Since the ordinary differential equations describing the missile trajectory in the atmosphere are not solvable in closed form, as they were in the case of an elliptical trajectory for a missile in a vacuum, these equations must be numerically integrated until the resulting trajectory pierces the Earth's surface. The numerical method currently being used is a fourth-order Runge-Kutta (RK) method with approximately ten steps per trajectory length or a second-order RK method with approximately twenty steps per trajectory length. The fourth-order RK method is used whenever the ballistic coefficient β of the missile is greater than 200 lbs/ft^2, whereas the second-order RK method is used whenever β is less than or equal to 200 lbs/ft^2. In either case, approximately forty evaluations of the equations of motion are required per drag IIP computation. Since the Runge-Kutta method is a single step method, the step size at each integration step is independent of the step size at all previous steps. The step size used is adjusted at each step so that it decreases as drag increases, in order to obtain a more accurate drag IIP in the same number of integration steps. A derivation of the equations of motion and the method of solution is described in [2], whereas the step size adjustment is described in [3].

2. Statement of Problem. Some missions at WSMR involve several simultaneous missiles. A drag IIP computation is required for each such missile. Experimental runs have indicated that only one drag IIP could be computed at a ten per second rate and at most four drag IIPs at a five per second rate [4]. Until recently drag IIPs were computed at a five per second rate, with a linear extrapolation of the last two computed drag IIPs being used to approximate the drag IIP at the next intermediate time. Additional mission requirements have necessitated the implementation of a variable rate for computing drag IIPs, ranging from ten per second to two per second [4]. At a two per second rate, four drag IIP approximations via linear extrapolation are required for each computed drag IIP, in order to output IIPs at a ten per second rate. Consequently, the output drag IIPs at a two per second rate are not as smooth and accurate as they are at a five or ten per second rate.

The purpose of this paper is to present an alternative method of computing drag IIPs and alternative methods of obtaining approximations to the drag IIPs at intermediate times, in order to decrease computation time and/or increase accuracy. In order to put this paper in its proper perspective, we briefly mention some previous investigations toward achieving these goals.

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3. Previous Investigations. In 1965 a method for obtaining a drag-corrected Kepler IIP for Athena missiles was investigated [5]. Briefly this method consisted of using position and velocity data from a previous Athena mission to calculate, in non-real time, a table of differences between Kepler IIPs and drag-corrected IIPs as a function of velocity. During the next Athena mission a linear extrapolation of this data was used to obtain a drag-corrected Kepler IIP in real time.

In 1975 various numerical integration methods for computing drag IIPs using digital and/or analog computers were investigated [6], which included Adams-Moulton, Milne-Hamming, Euler, a variable order Adams method called DIFSUB developed by C. W. Gear [7, 8, 9], and 2nd, 3rd, and 4th order Runge-Kutta methods. The conclusions reached were that the RK methods were better than the other methods, except possibly the method of Gear. Large errors were associated with the analog solutions.

Subsequently, alternative methods for expressing the equations of motion, using Encke's method [10, pp. 29-35] and two different versions of the method of variation of parameters [10, pp. 116-120 and 11], were investigated, as well as alternative numerical integration methods, such as an improved variable order, variable step Adams method [12], a rational extrapolation method [13, 14, 15, 16], and a Gauss-Jackson (Σ^2) method [10]. The conclusions drawn were that the method of variation of parameters took about twice as long as Cowell's method for the computation of the same drag IIPs, whereas Encke's method took about four times as long [17, p. 131 and 18, p. 15]. Furthermore, the alternative numerical integration methods investigated offered little if any improvement over the variable step RK method currently being used [17, p. 134].

Recently, an "f and g series" impact predictor algorithm, which is based upon a Taylor-series-in-time representation of a missile's position and velocity, was developed for which a "two to ten fold reduction in computer execution time for satellite orbits and a seven to ten fold reduction in execution time for ICBM trajectories" could be achieved over conventional numerical integration [19, p. 59]. A second report uses the f and g series technique to determine the "geographical distribution of debris impact coordinates that would result if the missile were destroyed," [20, p. 287]. This technique is an extension of the classical f and g series used in the solution of Kepler's equations of motion [21, pp. 107-111].

Such a Taylor series type of solution to an initial value problem is "generally impractical from a computational point of view," [22, p. 365]. In fact "... the necessity of calculating the higher derivatives makes Taylor's algorithm completely unsuitable on high-speed computers for general integration purposes," [23, p. 330]. However, in "comparison with fourth-order predictor-corrector and Runge-Kutta methods, the Taylor series method can achieve an appreciable saving in computer time, often by a factor of 100," [24, p. 389]. In view of these statements, the equations necessary to implement the Taylor series method are derived in this paper in order to determine whether or not the claims made for satellite orbits and ICBM trajectories are equally valid for short and medium range trajectories, such as those experienced by the missiles tested at WSMR.

4. Equation of Motion. The vector equation of motion of a missile is

$$\ddot{\bar{r}} = \ddot{\bar{r}}_u + \ddot{\bar{r}}_g + \ddot{\bar{r}}_d \ , \tag{4.1}$$

where $\ddot{\bar{r}}$ is the total acceleration of the missile, $\ddot{\bar{r}}_u$ is the unperturbed (Keplerian) acceleration, $\ddot{\bar{r}}_g$ is the perturbative acceleration due to higher order harmonics of the Earth's gravitational field, and $\ddot{\bar{r}}_d$ is the perturbative acceleration due to the Earth's atmospheric drag.

Missiles which are launched from one end of WSMR and which impact at the other end do not travel more than 143 miles, whereas missiles which are launched from Green River, Utah and impact on WSMR do not travel more than 500 miles. For such short and medium range missiles the perturbative acceleration $\ddot{\bar{r}}_g$ is insignificant compared with $\ddot{\bar{r}}_d$. Therefore, $\ddot{\bar{r}}_g = \bar{0}$ in this paper.

The expression for $\ddot{\bar{r}}_u$ is

$$\ddot{\bar{r}}_u = (-\mu/r^3)\bar{r} \ , \tag{4.2}$$

where $\bar{r} = (x, y, z)$ is the position vector of the missile, $r = (\bar{r}\cdot\bar{r})^{1/2}$

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is the distance of the missile from the origin of an inertial rectangular coordinate system, and μ is a gravitational constant. If r is measured in feet and time in seconds, then $\mu = 1.406559714 \times 10^6$ ft^3/sec^2.

The coordinate system used is right-handed, with origin at the Earth's center (geocentric), x-y plane in the equatorial plane, z-axis positive through the South Pole, and the x-axis aligned at 106°20' west longitude at the instant missile position and velocity data is obtained. Since this data is obtained in a non-inertial (relative) coordinate system which rotates with the Earth, the velocity components $\dot{x}_r$, $\dot{y}_r$, and $\dot{z}_r$ of the relative velocity vector $\bar{v}_r$ must be converted to corresponding components $\dot{x}$, $\dot{y}$, and $\dot{z}$ of the inertial velocity vector $\dot{\bar{r}}$ by

$$(4.3) \qquad \dot{x} = \dot{x}_r + \omega y, \quad \dot{y} = \dot{y}_r - \omega x, \quad \dot{z} = \dot{z}_r ,$$

where $\omega = 7.29211583 \times 10^{-5}$ rad/sec is the Earth's angular rate of rotation.

The vector acceleration for atmospheric drag is given by

$$(4.4) \qquad \ddot{\bar{r}}_d = -(\rho(h) v_r / 2\beta)\bar{v}_r ,$$

where $\bar{v}_r$ is the velocity of the missile relative to the Earth's atmosphere in ft/sec, $v_r = (\bar{v}_r \cdot \bar{v}_r)^{1/2}$, β is the ballistic coefficient of the missile in lbs/ft^2, and $\rho(h)$ is the density of the atmosphere in $slugs/ft^3$ at the current missile altitude h. The Earth's atmosphere is assumed to rotate with the Earth, the effects due to the wind are neglected, and β is assumed to be a constant for each missile.

If $\dot{\bar{r}}$ denotes the velocity of the missile in an inertial coordinate system and $\bar{\omega} = (0, 0, -\omega)$, then the relative velocity $\bar{v}_r$ is given by

$$(4.5) \qquad \bar{v}_r = \dot{\bar{r}} - \bar{\omega} \times \bar{r} .$$

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The atmospheric density is approximated by

(4.6) $$\rho(h) = A_0 \exp(A_1 h + A_2 h^2) ,$$

where $A_0 = 8.1283549 \times 10^{-2}$, $A_1 = -3.0319838 \times 10^{-5}$, and $A_2 = -1.6214665 \times 10^{-10}$ for $h < 70{,}000$ ft; $A_0 = 1.7237156 \times 10^{-1}$, $A_1 = -5.4682878 \times 10^{-5}$, and $A_2 = 3.7544187 \times 10^{-11}$ for $70{,}000 \text{ ft} < h < 150{,}000$ ft; and $A_0 = 0$ for $h > 150{,}000$ ft. The altitude h is given by

(4.7) $$h = r - R + 4000 ,$$

where $R = 20929831.0 - 71303.68411(z/r)^2$ (cf. [1, p. 41]) is the Earth radius of the Clarke Spheroid of 1866 at the same latitude as the missile but at the 4000 ft altitude of WSMR.

5. Taylor Series Method. The Taylor series expressions the missile's position $\bar{r} = \bar{r}(t)$ and velocity $\dot{\bar{r}} = \dot{\bar{r}}(t)$ at time t are

(5.1) $$\bar{r} = \Sigma_{i=0}^{n} (\tau^i/i!)\bar{r}_0^{(i)} + \bar{E}_1(t) ,$$

(5.2) $$\dot{\bar{r}} = \Sigma_{i=0}^{n-1} (\tau^i/i!)\bar{r}_0^{(i+1)} + \bar{E}_2(t) ,$$

where $\tau = t - t_0$ is the time interval or step size for re-initialization of the series and $\bar{r}_0^{(i)} = \bar{r}^{(i)}(t_0)$ is the i^{th} time derivative of the position evaluated at epoch t_0. The error vectors $\bar{E}_1(t)$ and $\bar{E}_2(t)$, due to the truncation of these Taylor series at the $\bar{r}_0^{(n)}$ term, are given by

(5.3) $$\bar{E}_1(t) = (\tau^{n+1}/(n+1)!)\ \bar{r}^{(n+1)}(\xi_1), \quad t_0 \leq \xi_1 \leq t,$$

(5.4) $$\bar{E}_2(t) = (\tau^n/n!)\ \bar{r}^{(n+1)}(\xi_2) , \quad t_0 \leq \xi_2 \leq t.$$

Following a rule of thumb, stated by Moore [25], of choosing n to be approximately equal to the number of significant decimal digits that can be carried by the computer, we choose n to have a maximum value

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of 6. However, for small values of the ballistic coefficient β, smaller values of n with smaller step sizes τ generally are preferred.

The derivatives $\bar{r}_0^{(i)}$ in (5.1) and (5.2) are obtained by differentiating $\ddot{\bar{r}}_u$ in (4.2) and $\dot{\bar{r}}_d$ in (4.4) with respect to time and adding. Hence from (4.1), with $\dot{\bar{r}}_g = \bar{0}$,

$$\text{(5.5)} \qquad d^m \ddot{\bar{r}}/dt^m = d^m \ddot{\bar{r}}_u/dt^m + d^m \dot{\bar{r}}_d/dt^m.$$

In the development of the f and g series algorithm in [19], the expressions for the second and higher derivatives of $\ddot{\bar{r}}_u$ failed to include the acceleration $\dot{\bar{r}}_d$ and its derivatives [19, p. 23, (6)], resulting in the omission of a term in the expression for $d^2\ddot{\bar{r}}/dt^2$ [20, p. 301, (15)], and the omission of terms in all higher derivatives of $\ddot{\bar{r}}$. Therefore, it was decided to determine these higher derivatives of $\ddot{\bar{r}}_u$ and $\dot{\bar{r}}_d$ independently from [19] and [20], in order to correct this omission.

Applying Leibnitz' Theorem for finding the n^{th} derivative of the product of two functions to (4.2), we obtain

$$\text{(5.6)} \qquad \frac{d^m \ddot{\bar{r}}_u}{dt^m} = -\mu \sum_{k=0}^{m} \binom{m}{k} \frac{d^k}{dt^k}\left(\frac{1}{r^3}\right) \frac{d^{m-k}\bar{r}}{dt^{m-k}} ,$$

in which

$$\text{(5.7)} \qquad \begin{cases} d(1/r^3)/dt = 3\dot{r}/r^4 , \\ d^2(1/r^3)/dt^2 = 3\ddot{r}/r^4 - 4\dot{r}^2/r^5 , \\ d^3(1/r^3)/dt^3 = 3\dddot{r}/r^4 - 36\,\dot{r}\ddot{r}/r^5 + 60\,\dot{r}^3/r^6 , \\ d^4(1/r^3)/dt^4 = 3r^{iv}/r^4 - 48\,\dot{r}\dddot{r}/r^5 - 36\,\ddot{r}^2/r^5 \\ \qquad + 360\,\dot{r}^2\ddot{r}/r^6 - 360\,\dot{r}^3/r^7 . \end{cases}$$

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Since $\dot{r} = (\dot{\bar{r}} \cdot \dot{\bar{r}})^{1/2}$, we easily obtain

$$(5.8) \quad \begin{cases} \ddot{r} = (\dot{\bar{r}} \cdot \ddot{\bar{r}}) \,/\, \dot{r} \,. \\ \dddot{r} = (-\ddot{r}^2 + \ddot{\bar{r}} \cdot \ddot{\bar{r}} + \dot{\bar{r}} \cdot \ddot{\bar{r}}) \,/\, \dot{r} \,, \\ r^{iv} = (-3\,\ddot{r}\;\dddot{r} + 3\,\ddot{\bar{r}} \cdot \dddot{\bar{r}} + \dot{\bar{r}} \cdot \bar{r}^{iv}) \,/\, \dot{r} \,. \end{cases}$$

The corresponding higher derivatives of $\dot{\bar{r}}_d$ are determined by applying Leibnitz' Theorem to (4.4), to obtain

$$(5.9) \qquad \frac{d^m \dot{\bar{r}}_d}{dt^m} = -\frac{1}{2\beta} \sum_{k=0}^{m} \binom{m}{k} \frac{d^k(\rho v_r)}{dt^k} \frac{d^{m-k} v_r}{dt^{m-k}} \,,$$

where

$$(5.10) \qquad \frac{d^k(\rho v_r)}{dt^k} = \sum_{i=0}^{k} \binom{k}{i} \frac{d^k \rho}{dt^k} \frac{d^{k-i} v_r}{dt^{k-i}} \,.$$

From (4.5), we have

$$(5.11) \qquad d^m \bar{v}_r / dt^m = d^m \dot{\bar{r}} / dt^m - \bar{\omega} \times d^m \bar{r} / dt^m \,.$$

Since $v_r = (\bar{v}_r \cdot \bar{v}_r)^{1/2}$, we easily obtain

$$(5.12) \quad \begin{cases} \dot{v}_r = (\bar{v}_r \cdot \dot{\bar{v}}_r) \,/\, v_r \,, \\ \ddot{v}_r = (-\dot{v}_r^{\,2} + \dot{\bar{v}}_r \cdot \dot{\bar{v}}_r + \bar{v}_r \cdot \ddot{\bar{v}}_r) \,/\, v_r \,, \\ \dddot{v}_r = (-3\dot{v}_r\,\ddot{v}_r + 3\dot{\bar{v}}_r \cdot \ddot{\bar{v}}_r + \bar{v}_r \cdot \dddot{\bar{v}}_r) \,/\, v_r \,, \\ v_r^{iv} = (-4\dot{v}_r\,\dddot{v}_r - 3\ddot{v}_r^{\,2} + 3\ddot{\bar{v}}_r \cdot \ddot{\bar{v}}_r + 4\dot{\bar{v}}_r \cdot \dddot{\bar{v}}_r + \bar{v}_r \cdot \bar{v}_r^{iv}) \,/ v_r . \end{cases}$$

The higher time derivatives of ρ in (5.10) are obtained by repeated applications of the chain rule of differential calculus to $\rho(h(t))$ in (4.6). We first note from (4.7) that the time derivatives of the Earth's radius R are small compared with the time derivatives of $r = \bar{r}$. Therefore, we have

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(5.13) $$d^m h/dt^m = d^m r/dt^m + \varepsilon_m ,$$

where $|\varepsilon_m| << |d^m r/dt^m|$.

Applying the chain rule to $\rho(h(t))$ and using (5.13), we obtain

$$d\rho/dt = \dot{r}\partial\rho/\partial h ,$$

(5.14) $$\begin{cases} d^2\rho/dt^2 = \dot{r}^2\partial^2\rho/\partial h^2 + \ddot{r}\ \partial\rho/\partial h , \\ d^3\rho/dt^3 = \dot{r}^3\partial^3\rho/\partial h^3 + 3\ \dot{r}\ \ddot{r}\ \partial^2\rho/\partial h^2 + \dddot{r}\ \partial\rho/\partial h , \\ d^4\rho/dt^4 = \dot{r}^4\partial^4\rho/\partial h^4 + 6\ \dot{r}^2\ddot{r}\ \partial^3\rho/\partial h^3 \\ \qquad + (3\ \ddot{r}^2 + 4\ \dot{r}\ \dddot{r})\partial^2\rho/\partial h^2 + r^{iv}\ \partial\rho/\partial h . \end{cases}$$

Differentiating (4.6), we have

(5.15) $$\begin{cases} \partial\rho/\partial h = (A_1 + 2A_2h)\rho , \\ \partial^2\rho/\partial h^2 = 2A_2\rho + (A_1 + 2A_2h)\partial\rho/\partial h , \\ \partial^3\rho/\rho h^3 = 4A_2\partial\rho/\partial h + (A_1 + 2A_2h)\ \partial^2\rho/\partial h^2 , \\ \partial^4\rho/\partial h^4 = 6A_2\partial^2\rho/\partial h^2 + (A_1 + 2A_2h)\partial^3\rho/\partial h^3 . \end{cases}$$

All quantities required for the computation of the derivatives $\bar{r}_0^{(i)}$, $i = 1, \ldots, 6$, in the Taylor series (5.1) and (5.2) can now be determined from the preceding equations. To implement the Taylor series method, a value of n is choosen between 2 and 6 with lower values corresponding to smaller values of β. The step size τ is determined such that the maximum number of steps permitted per trajectory integration is not exceeded, and such that the local truncation errors $\bar{E}_1$ in (5.3) and $\bar{E}_2$ in (5.4) are not exceeded. Therefore, given the position $\bar{r}_0$ and velocity $\dot{\bar{r}}_0$ of a missile at epoch t_0, we obtain the position $\bar{r}$ and velocity $\dot{\bar{r}}$ of a missile at time t from (5.1) and (5.2). This process is repeated until impact time T, for which $r(T) = R(T)$.

Since the preceding computations were made in an inertial coordinate system, the true impact coordinates x_I, y_I, and z_I can

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be obtained from the coordinates x_T, y_T, and z_T of the position vector $\bar{r}(T)$ at impact by a rotation through ωT radians as follows

$$(5.16)\quad \begin{cases} x_I = x_T \cos \omega T - y_T \sin \omega T , \\ y_I = x_T \sin \omega T + y_T \cos \omega T , \\ z_I = z_T . \end{cases}$$

6. Methods for Approximating Drag IIPs between Computational Cycles. Ten times a second, or once every 100 milliseconds, all data on a missile, such as its position, velocity, and drag IIP, is updated. However, the computation of drag IIPs for several missiles cannot be completed in fewer than 100 milliseconds even using the Taylor series method, because much of the computer's time is spent making many other computations during each 100 millisecond time period. For example, if drag IIPs can be computed only once every 500 milliseconds, then approximations to the drag IIPs are required four times per 500 millisecond computational cycle. The current method for obtaining these approximations is by a linear extrapolation of previously computed drag IIPs, considered as functions of range time. Unfortunately, this method does not account for new values of the missile's initial position $\bar{r}_0$ and velocity $\dot{\bar{r}}_0$ at these intermediate times and is much less accurate than the following improved methods.

The first improved method for approximating drag IIPs between computational cycles is by a quadratic extrapolation of the components of previously computed drag IIPs, considered as functions of their corresponding components of the vacuum IIPs. Since vacuum IIPs are always computed every 100 milliseconds anyway, no additional computer time is required for their use in this method. In fact, it requires about the same amount of computational time as the current linear extrapolation method (one millisecond), yet is more accurate by an average factor of 13. Furthermore, it does account for new intermediate time values of the missile's initial position $\bar{r}_0$ and velocity $\dot{\bar{r}}_0$ since the vacuum IIPs at these times are functions of $\bar{r}_0$ and $\dot{\bar{r}}_0$. However, if consecutive pairs of components of vacuum IIPs are "close" together, then this quadratic extrapolation method may yield erroneous drag IIP approximations. Therefore, the following method which avoids this problem is recommended.

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The second improved method for approximating drag IIPs between computational cycles is by a quadratic extrapolation of the differences between previously computed drag IIPs and their corresponding vacuum IIPs, considered as functions of range time. It too requires about the same amount of computational time as the current linear extrapolation method, yet is more accurate by an average factor of 6.

In general, higher order extrapolations may yield increasingly worse approximations to the drag IIPs as the order is increased. This would be especially true if the radar-determined position and velocity of the missile were not following a smooth trajectory, in which case an accurate drag IIP would be needed most. In fact, "If polynomial extrapolation must be done with poorly behaved functions, then very low degree extrapolation is usually the safest, but even this should be carried out only for values of x very close to the tabulated region," [26, p. 58].

7. Conclusions. The use of the Taylor series method resulted in equivalent drag IIPs being computed in two to ten times less time than by the currently used Runge-Kutta method. The use of the method of quadratic extrapolation of the differences between previously computed drag IIPs and their corresponding vacuum IIPs resulted in approximations of drag IIPs between computational cycles being computed over six times more accurately than by the currently used linear extrapolation method, with about the same amount of computational time, and without the possibility of erroneous approximations.

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